

**Exercise 13**

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(x) = \int_1^{e^x} \ln t \, dt$$

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**Solution**

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx} \int_a^x f(t) \, dt = f(x).$$

In order to make the upper limit a single variable, let  $u = e^x$ .

$$h(x) = \int_1^u \ln t \, dt$$

As a result, using the chain rule,

$$\begin{aligned} h'(x) &= \frac{d}{dx} \int_1^u \ln t \, dt \\ &= \frac{du}{dx} \frac{d}{du} \int_1^u \ln t \, dt \\ &= \frac{du}{dx} (\ln u) \\ &= e^x (\ln e^x) \\ &= e^x (x \ln e) \\ &= x e^x. \end{aligned}$$